DISPERSION RELATION FOR NONLINEAR WAVES IN A FLUID WITH GAS BUBBLES

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A survey of the research devoted to studying the propagation of small perturbations in a fluid with gas bubbles is given in [1]. Obtained in these papers, in particular, are dispersion relations; the influence of dissipative mechanisms as well as diverse factors such as polydisperseness of the mixture, polymer admixtures in the fluid, etc., is clarified. The investigations mentioned were performed on the basis of the linearized equations of motion. Meanwhile, it is known that a fluid with gas bubbles is a nonlinear dispersing medium. Consequently, a dispersion relation for nonlinear waves is obtained in this paper in the absence of dissipation, and the frequency and amplitude dependence of the phase velocity are computed for waves passing through a given equilibrium state of the medium.

According to [2, 3], stationary nonlinear waves in a fluid with gas bubbles with hydrodynamic nonlinearity, the nonlinearity of the radial fluid motion around the bubbles, and the nonlinearity of the fluid equation of state taken into account are described by solutions of the equation

$$\left(\frac{dp}{d\eta}\right)^2 = -\frac{2\rho^2 (V(p))^{1/3} (V_p)^{-2} (H(p) + H_0)}{(1 + m\rho V(p))^2 m C_1^2 \gamma p_0} \equiv F(p; D, H_0),$$

where $mV = C_2/C_1^2 - p/C_1^2 - (1 + n(p - p_0))^{-1/n}$; $H = -\frac{m p_0}{\gamma - 1} V^{1-\gamma} + \frac{p^2}{2C_1^2} - \frac{1 - np_0 + p}{\rho(n-1)}$; $H_0 = \frac{C_3}{C_1} - \frac{C_2^2}{2C_1^2}$; H_0 is an amplitude parameter, C_1, C_2, C_3 are dimensionless mass, momentum, and energy fluxes of the combined phase deformation, p_0', ρ_0', c_0 are equilibrium values of the pressure, density, and speed of sound in the fluid, R_0, K_0 are equilibrium values of the bubble radius and volume concentration of the gas, γ, n are the adiabatic indices for the gas and the fluid. Introduced here are the dimensionless variables $\eta = x - Dt$, $D' = Dc_0$, $x = x'\omega_0/c_0$, $t = t'\omega_0$, $V = (R/R_0)^3$, $p' = p\rho_0' c_0^2$, $p_0' = p_0 \rho_0' c_0^2$, $m = K_0/(1 - K_0)$, $\rho^n = 1 + n(p - p_0)$, $\omega_0^2 = 3\gamma p_0'/\rho_0' R_0^2$, $C_1^2 = D^2(1 - K_0)^2$, $C_2 = p_0 + (1 - K_0)D^2$.

Following [4, 5], we find the length of the nonlinear periodic wave by analogy with linear theory:

$$\lambda = \lambda(D, H_0) = 2\int_{p_1}^{p_2} dp/\sqrt{F(p; D, H_0)}$$

(p_1 and p_2 are the minimal and maximal pressures in the wave); then the wave number and frequency are determined in the usual manner: $k = 2\pi/\lambda$, $\omega/\omega_0 = Dk(D, H_0)$. This is indeed the dispersion relation for nonlinear waves which we represent as

$$\frac{\omega}{\omega_0} = \pi D \left(\sqrt{\frac{C_1^2 m\gamma p_0}{2}} \int_{p_1}^{p_2} \sqrt{\frac{V^{-1/3} V_p^2}{H(p) + H_0}} \frac{1 + m\rho V}{\rho} dp \right)^{-1}. \quad (1)$$

In the linear case the periodic solution is $p - p_0 = a \cos \theta$, and (1) goes over into the known dispersion relation for linear waves:

$$\left(\frac{\omega}{\omega_0}\right)^2 = \frac{c_f^2(p_0)\left(D^2 - c_e^2(p_0)\right)}{c_e^2(p_0)\left(D^2 - c_f^2(p_0)\right)}, \quad (2)$$

where c_e and c_f are the equilibrium and frozen speeds of sound. The dispersion relation (1) has an implicit form and shows an especially nonlinear dependence of the frequency on the phase velocity and amplitude. To carry out a quantitative analysis we limit ourselves

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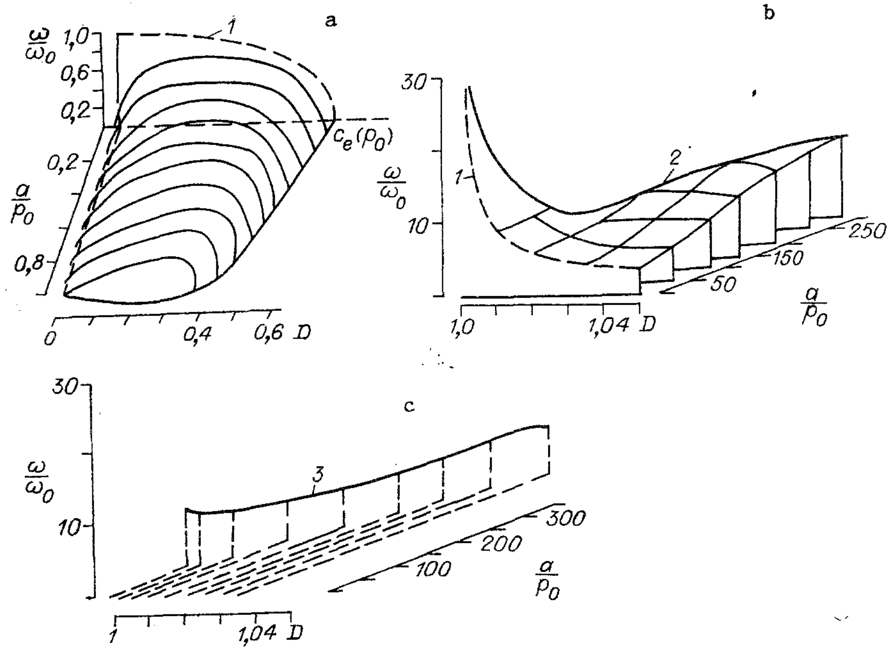

Fig. 1

to the construction of the nonlinear dependence (1) for waves passing through a given equilibrium state: $p_0' = 10^5$ Pa, $K_0 = 10^{-4}$, $c_0 = 1.5\ 10^3$ m/sec ($\gamma = 1.4$, $n = 7.15$).

We note first that for a fixed double wave amplitude $p_2 - p_1 = a$, the specific volume of the mixture $v = v_0^2(p_0 - p)/D^2 + v_0$ grows without limit for $p \cong p_1$ as $D^2 \to 0$ ($p_2 \to p_0$, $p_1 \to p_0 - a$). Consequently, although for any fixed amplitude as $D \to 0$ $\omega/\omega_0 \to 0$, the solution starting with a certain critical value of $D_*(a)$ actually emerges outside the scope of the constraints of the model under consideration.

The results of a computation of (1) are presented in the figure where a is the frequency and amplitude dependence of the phase velocity of the waves whose velocity is less than $c_e(p_0)$. According to the computation, in the wave velocity range under consideration, for which the relative flow of the medium is subsonic ($(u - D)^2 < c_f^2(p)$), the wave frequency diminishes as the amplitude increases at a fixed velocity; b is the dependence of the phase velocity on the frequency and amplitude for waves whose velocity is greater than $c_f(p_0)$. The far boundary of the dispersion surface (curve 2) corresponds to waves whose amplitude is maximal but does not exceed the limit from [2, 3]. According to the computation, for the waves under consideration for which the relative flow of the medium is supersonic, the wave frequency grows with the increase in amplitude for a fixed velocity. The existence of periodic waves in which the relative flow of the medium goes over continuously from the supersonic to the subsonic regime and conversely is shown in [2]. It turns out here that these waves passing through a given equilibrium state are propagated at velocities exceeding $c_f(p_0)$. For convenience, the curve 3 in the figure displays the dependence between the frequency, amplitude, and velocity of such substantially nonlinear waves. In contrast to waves in which the relative flow of the medium is supersonic, the passage to the limit as $D \to c_f(p_0)$ holds in the case being considered, i.e., as D diminishes, the point (D, ω/ω_0, a/p_0) of the curve 3 tends to the point ($c_f(p_0)$, $\tilde{\omega}/\omega_0$, $\tilde{a}/p_0$) that corresponds to a wave being propagated at the frozen speed of sound. The relative flow of the medium in the latter is subsonic but $\tilde{\omega}/\omega_0 \approx 8$, $\tilde{a} = 113p_0$.

In the velocity range ($c_e(p_0)$, $c_f(p_0)$) the stationary perturbations are solutions for which the dispersion relation degenerates into a nonlinear dependence between the velocity and the amplitude. The dispersion dependence (2) is shown by line 1 for comparison.

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STABILITY OF A DUSTY NONISOTHERMAL GAS JET

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Interest in modeling the behavior of gas-dispersed flows with large parameter gradients has increased greatly in recent years for several reasons. On the one hand, there are an increasing number of practical uses for such flows. Examples of this are found in the chemical industry and in the area of environmental protection (propagation of aerosols). On the other hand, the interest also stems from improved possibilities for calculating such flows. In this regard, investigators are especially attracted to the problem of the stability of gas-dispersed flows. The solution of this problem would in several cases make it possible to obtain estimates of the critical parameters corresponding to the transition from laminar to turbulent flow. Calculations of stability performed in [1-3] for dusty isothermal gas flows showed that a flow may be appreciably stabilized by particles (the critical Reynolds numbers may increase by several orders of magnitude under certain conditions). No calculations have been made of the stability of thermally stratified gas flows with a disperse phase, although such calculations would most likely have practical value. Here, we examine the stability of a dusty plane jet with a temperature differing considerably from the medium in which the jet is flowing.

The flow of a submerged viscous nonisothermal gas-dispersed jet is described by the system of Navier–Stokes equations with allowance for the gas-particle interaction, which is modeled by a term of the Stokes force type. As was noted in [1-3], an important parameter is $\beta = \tau/\tau_0$, where $\tau = L/(U_m \alpha C)$ (L and U_m are the characteristic scales of length and velocity of the jet, while α and C are the wave number and the phase velocity of the perturbations). The quantity $\tau_0 = \rho_0 d^2(18\mu_g)$ is the time of Stokes relaxation relative to the particle velocity (ρ_0 is the density of the particle material, d is the particle diameter, and μ_g is the viscosity of the gas). The case $\beta \ll 1$ is usually realized in actual dusty flows. The following evaluations can serve as an illustration. For particles of the diameter 10^{-4} m and density $\rho \approx 10^4$ kg/m³ with a hot-air viscosity $\mu_g \approx 2\cdot10^{-5}$ kg/(m·sec), the relaxation time is $\tau_0 \approx 5/18$ sec. At the same time, for typical jet scales $L \approx 10^{-2}$ m, $U_m \approx 2\cdot10^2$ m/sec, and $\alpha C \approx 10^{-2}$ (from the results of our study), $\tau \approx 5\cdot10^{-3}$ sec and $\beta \approx 18\cdot10^{-3}$. Thus, the charcteristic fluctuation velocities of the particles are considerably less than the fluctuation velocity of the gas. As a result, in the analysis of stability presented here, the disturbance of the particles can be ignored. Since the parameter $\beta_1 = 18\rho_g L^2/(\rho_0 \mathrm{Re}\, d^2)$ depends on the Reynolds number $\mathrm{Re} = LU_m/\nu_g$ ($\nu_g^{-1} = \rho_g/\mu_g$, ρ_g is the density of the gas), it is convenient to use it as an independent varible (in [2-4], β was assigned; this led to obvious problems in calculating neutral curves with $\beta_1 \ll 1$) . As the characteristic parameter in the present study, we take $A = 18\delta(L/d)^2$ (δ is the volumetric concentration of particles). For the above parameters, $18(L/d) \approx 1.8\cdot10^5$ and with a change in δ from 10^{-5} to 10^{-2}, A may incrase from 1.8 to $1.8\cdot10^3$.

Proceeding on the basis of the Navier–Stokes equations for a nonisothermal flow and using Stokes' law to describe the effect of the particles on the gas flow, we can obtain the following system of equations:

$$\rho\frac{dU}{dt} = -\frac{\partial P}{\partial x} + \frac{2}{\mathrm{Re}_-}\frac{\partial}{\partial x}\left(\mu\frac{\partial U}{\partial x}\right) + \frac{1}{\mathrm{Re}_-}\frac{\partial}{\partial y}\left[\mu\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)\right] - \frac{2}{3\mathrm{Re}_-}\frac{\partial}{\partial x}\left[\mu\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\right] + \frac{An\mu}{\mathrm{Re}_-}(U - U_0),$$

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